De factoribus numerorum compositorum dignoscendis

Disquisitio Academica,

cujus partem tertiam,

adnuente amplissima Fac. Philos.

Præside

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pro summis in philosophia capessendis honoribus

exhibet

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(—9) est differentia primi et secundi; ideoque et quartum (19) erit differentia secundi (10) et tertii (—9). Est etiam 19 = 10—(—9), & — 28 = (—9) — 19; continua igitur subtractione reliqua residua inveniuntur. Nempe 47 = 19—(—28), —28—47 = —75 $\backsim$ 34, &c. (Vide schema a latere appositum *).

Inde vero ipsius 109 schemata characteristica eliguntur; $\begin{bmatrix} -II \\ 9\,. \end{bmatrix}$, $\begin{bmatrix} +III \\ 19\,. \end{bmatrix}$, $\begin{bmatrix} +VII \\ 13\,. \end{bmatrix}$, $\begin{bmatrix} -IX \\ 8\,. \end{bmatrix}$ &c.

Unumquodque quidem residuum schema suppeditat, majora vero (ut 47, 34), cum operationi faciliter instituendæ minus apta sint, omittuntur; et ipsorum quidem quodcunque suum α suppeditat, quæ ideo signis α_0, α_1, α_2, ... α_ν distingui possunt. Si igitur 10^ν per p divisum residua ministraverit α_0, α_1, α_2 &c... cum $\nu = 0$, 1, 2 &c. fuerit, ideoque universim $10^\nu \backsim \alpha_\nu$ fuerit, schema $\begin{bmatrix} +\nu \\ \alpha_\nu . \end{bmatrix}$ erit.

*) n	residua
0	1
1	10
2	—9
3	+19
4	—28
5	+47 (=19+28)
6	34 ($\backsim$ —28—47)
7	13 (=47—34)
8	+21
9	—8 &c.

His positis, de numeris (α_ν) hoc tenendum est: *Si* $\begin{bmatrix} +\nu \\ \alpha_\nu . \end{bmatrix}$ *et* $\begin{bmatrix} +n \\ \alpha_n . \end{bmatrix}$ *duo schemata (diversa vel etiam eadem si placet) ejusdem divisoris* (p) *fuerint, etiam* $\begin{bmatrix} +u \\ \alpha_u . \end{bmatrix}$ *ejusmodi erit, si nempe fuerit* $u = n + \nu$, *atque* $\alpha_\nu . \alpha_n \underset{p}{\backsim} \alpha_u$. Est enim $\alpha_\nu \underset{p}{\backsim} 10^\nu$, et $\alpha_n \underset{p}{\backsim} 10^n$, ideoque $\alpha_\nu . \alpha_n \underset{p}{\backsim} 10^{n+\nu} \underset{p}{\backsim} 10^u \underset{p}{\backsim} \alpha_u$, cum $n + \nu = u$ sit.

Obs. Similiter calculi compendium, modo observatum, facile demonstratur. Si nempe fuerit ex. gr. $\alpha_s \underset{p}{\backsim} b\alpha_r + a\alpha_6$, fore uni-

versim dicimus $\alpha_{s+n} \underset{p}{\backsim} b \cdot \alpha_{r+n} + a \cdot \alpha_n$. Quoniam enim $\alpha_s \underset{p}{\backsim} 10^s$, et $\alpha_r \underset{p}{\backsim} 10^r$, erit $10^s \underset{p}{\backsim} b \cdot 10^r + a$, ideoque, in 10^n ducendo, $10^{s+n} \underset{p}{\backsim} b.10^{r+n} + a.10^n$, seu, cum $10^{s+n} \underset{p}{\backsim} \alpha_{s+n}$, $10^{r+n} \underset{p}{\backsim} \alpha_{r+n}$ et $10^n \underset{p}{\backsim} \alpha_n$ sit, $\alpha_{s+n} \underset{p}{\backsim} b \cdot \alpha_{r+n} + a \cdot \alpha_n$. I. qu. e. d.

Ex. gr. sit $p=17$; erit $\left\{ \begin{matrix} \nu & = 0, & 1, & 2, & 3. \text{ \&c.} \\ \alpha_\nu & = 1, & 10, & 15, & 14 \text{ \&c.} \end{matrix} \right\}$, ideoque $\alpha_3 = \alpha_2 - \alpha_0$, et universim $\alpha_{\nu+3} = \alpha_{\nu+2} - \alpha_\nu$, nempe $\alpha_4 = 14 - 10 = 4$, $\alpha_5 = 4 - 15 \underset{17}{\backsim} 6$; &c.

Ex. $p = 149$: $\left\{ \begin{matrix} \nu & = 0, & 1, & 2, & 3, & 4 & \text{\&c.} \\ \alpha_\nu & = 1, & 10, & 100, & 106, & 17 & \text{\&c.} \end{matrix} \right\}$.

At $106 = 100 + 6.1$, seu $\alpha_3 = \alpha_2 + 6\alpha_0$, ideoque generatim $\alpha_{n+3} = \alpha_{n+2} + 6\alpha_n$, et speciatim $106 + 6.10 \backsim 17 = \alpha_4$, et $17 + 6.100 \backsim 17 - 6.49 \underset{149}{\backsim} 21 = \alpha_5$. Quoniam vero $21 = 17 + 4.1$, erit $\alpha_{n+5} = \alpha_{n+4} + 4 . \alpha_n$, et speciatim $\alpha_6 = 21 + 40 = 61$, et similiter $\alpha_7 = 14$, $\alpha_8 = -9$. Jam vero $-9 = 1 - 10$, ideoque $\alpha_{n+8} = \alpha_n - \alpha_{n+1}$, quæ formula maxime idonea est, cujus ope reliqua residua calculentur. Differentia nempe duorum residuorum, sibi invicem subsequentium, octavo loco proximum suppeditat. Residua igitur ex divisore 149 (in 10^n) sunt

α_0 = 1, 10, 100, 106, 17, 21, 61, 14 (—9, etc.,)
α_8 =—9,—90, —6, 89, —4,—40, 47, 23; (81 = α_{16})
α_{16} = 81,—84,—95, 93, 36,—87, 24,—58;
α_{24} = 16, 11, 110, 57, 123, 38, 82,—74;
α_{32} = 5,—99, 53,—66, 85,—44, 7 &c.

Indeque schemata eliguntur commoda:

$\left[{}^{-VIII}_{9\ \cdot}\right]$, $\left[{}^{-X}_{6\ \cdot}\right]$, $\left[{}^{-XII}_{4\ \cdot}\right]$, $\left[{}^{+XXV}_{11\ \cdot}\right]$, $\left[{}^{+XXXII}_{5\ \cdot}\right]$, $\left[{}^{+XXXVIII}_{7\ \cdot}\right]$ &c...

quorum tamen pleraque non nisi ad magnos numeros, de factore 149 inquirendos, quadrent.

Obs. Simile compendium in decimalibus ipsius $\frac{1}{p}$ perquirendis adhiberi potest.

Sic ex. gr. cum $p = 109$, habetur $\frac{1}{p} = 0{,}009\,174312 - \frac{8}{p \cdot 10^9}$, at $-8 = 2 - 10$, et $\frac{10}{p} = 0{,}0917..$, atque $\frac{2}{p} = 0{,}018348624..$ quare

$$-\frac{8}{p} = \left\{\begin{matrix} +\,0{,}018348624.. \\ -\,0{,}09174312.. \end{matrix}\right\} = -\,0{,}073394..,$$ atque ergo

$$\frac{1}{p} = \left\{\begin{array}{l|l|l} 0{,}009174312 & 0183486 & 2\ldots \quad a) \\ & 0917431 & \ldots\ldots = b) \end{array}\right.$$

seu $\frac{1}{109} = 0{,}00917431 | 19266055 \; | 0\ldots\ldots = c)$, scilicet $c) = a) - b)$. Et hic computus quousque libet continuari potest, scribendo ulterius in fine seriei $a)$ duplum ipsius c, ipsamque numerorum seriem $c)$ in $b)$; nempe $2 | 01834862 | 38532110$

$$\begin{array}{l} -|0917431|1\;9266.. \quad \&c.. \\ \hline |..66055|0\;4586.. \end{array}$$

Et similiter in reliquis: simplex enim residuorum additio simplicem decimalium computum suppeditat. Quin etiam similis computandi modus ex simplici residuorum ratione elicitur. Sic. ex. gr. $10^4 \underset{109}{\backsim} 81$, atque $10^{11} \underset{109}{\backsim} 72$, et $81 : 72 = 9 : 8$; ideoque decimales in $\frac{10^4}{109}$ ad eos in $\frac{10^{11}}{109} = 9 : 8 = 1 : 1 - \frac{1}{9}$ et $\frac{1}{109} = 0{,}0091 + \frac{81}{109 \cdot 10^4} =$

$$= 0{,}0091\,|\,7431192 + \frac{72}{109 \cdot 10^{11}} = 0{,}0091743\,1192 + \frac{\overset{\times}{\overline{1 - \frac{1}{9}}} \cdot \frac{81}{109}}{10^{11}} =$$

$$= 0{,}0091743\,1192.. + \left[1 - \frac{1}{9}\right] . \overset{\times}{0}07431192.. \text{ seu}$$

$$\frac{1}{109} = 0{,}0091|743\overset{\times}{1}192|660550|$$
$$(743\ldots:9)\quad|\ \underline{825688|\qquad\qquad}$$

$\left[1-\frac{1}{9}\right]\cdot\frac{81}{109.10^{14}} = \quad +660550\ldots$ quo facto, divisio per 9 et subtractio iterum per sex locos decimales institui potest et $\frac{1}{109}$ sex signis augeri; idemque iterari, donec totus decimalium cyclus habeatur. Vel etiam cum $10^{11} \underset{109}{\backsim} 17 = \frac{1}{2}.34 \backsim \frac{1}{2}.10^6$ inveniatur, inde a loco XIX sola dimidiatione decimalium jam inventorum computus perficietur.

Hæc vero res elementa Arithmetices nimis forsan tangit, quominus diutius in ipsa morari liceat.

II) Deinde perquirendi sunt numeri β et ν, schemati $\begin{bmatrix}\nu\mp\\ .\ \beta\end{bmatrix}$, competentia. Hoc vero eodem redit, ut quæratur numerus formæ $\beta.10^{\nu}\mp 1$, qui multiplus dati numeri (p) sit.

Computus vero, cujus ope ejusmodi numerus invenitur, divisio est quasi reversa. Initium nempe ab ± 1 ducimus, cui multiplum numeri dati (p) addimus eum, qui summam per 10 divisibilem efficit; deinde, cifra (o) neglecta, idem cum reliquis perficimus. Sit ex. gr. $p = 109$: et invenientur ita residua reversa 11, 12, 23, 35, 58, (seu -51), 93 ($\backsim -16$) &c., quæ valorem ipsius β in schemate $\begin{bmatrix}\nu+\\ .\ \beta\end{bmatrix}$ præbent. Est enim $11.10^1 = 110 = 109+1$, seu $11.10^1 \backsim +1$, ideoque, schema $\begin{bmatrix}I+\\ .11\end{bmatrix}$; et similiter $12.10^2 = 1200 \backsim 110 \backsim +1$, et schema $\begin{bmatrix}II+\\ ..12\end{bmatrix}$; reliquæque $\begin{bmatrix}III+\\ .23\end{bmatrix}$, $\begin{bmatrix}IV+\\ .35\end{bmatrix}$, $\begin{bmatrix}V+\\ .58\end{bmatrix}$ $\begin{bmatrix}VI+\\ .93\end{bmatrix}$ seu $\begin{bmatrix}VI\ \ +\\ .(-16)\end{bmatrix}$ i. e. $\begin{bmatrix}VI\\ .\ \overline{16}\end{bmatrix}$; &c.

$$\begin{array}{r}
+1\\
\underline{+109}\\
11|0\\
\underline{+109|}\\
12|0\\
\underline{(2.109)=218|}\\
23|0\\
\underline{(3.109=)327|}\\
35|0\\
\underline{545|}\\
58|0\\
\underline{872|}\\
93|0
\end{array}$$

Si igitur modo dicto reverso numerus dividitur fictus formæ X oo..ooo1 per p, residua ita obtenta valores ipsius β, et numerus cifrarum (o) neglectarum exponentes (ν) in schemate $\begin{pmatrix} \nu+ \\ .\beta \end{pmatrix}$ præbebunt.

Sit ex. gr. $p = 149$; eruntque:

$\nu =$ I, II, III, IV, V, VI, VII, IIX, IX, X, XI.

$\beta = 15, 76, 97, 114, 71, 22, 32, 33, 48, 124, 72.$

seu $\beta = -73, -52, -35, \ldots\ldots\ldots\ldots -25,$

unde schemata eliguntur:

$\begin{bmatrix} I+ \\ .15 \end{bmatrix}, \begin{bmatrix} V+ \\ .71 \end{bmatrix}, \begin{bmatrix} VI+ \\ .22 \end{bmatrix}, \begin{bmatrix} VIII+ \\ .33 \end{bmatrix}, \begin{bmatrix} X- \\ .25 \end{bmatrix}$ &c...

```
0000001
  +149
   15|0
  +745
   76|0
  894
   97|0
 1043
  114|0
  596
   71|0 &c.
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Obs. Etiam hæc residuorum series eodem modo, ac in casu præcedenti, abbreviate continuari potest; posito nempe $\beta_0 = 1$, $\beta_1 = 15$, $(\beta_2 = 76)$ &c., erit, quoniam $76 = 5.15 + 1$, $\beta_2 = 5.\beta_1 + \beta_0$, et universim $\beta_{\overline{n+2}} = 5.\beta_{\overline{n+1}} + \beta_n$; Ex. gr. $\beta_9 = 5\beta_8 + \beta_7 = 5.33 + 32 = 197 \backsim 48$; et demonstratio eadem. Similiter, quoniam $33 = 32 + 1$, et $72 = 71 + 1$, erit universim $\beta_{n+8} = \beta_{\overline{n+7}} + \beta_n$ atque $\beta_{n+11} = \beta_{n+5} + \beta_n$, unde computus valde sublevatur. Item, quoniam $33 = 22 + \frac{1}{2}.22$, erit universim $\beta_{n+8} = \beta_{n+6} + \frac{1}{2}(\beta_{n+6})$ seu $\beta_{n+2} = \beta_n + \frac{1}{2}(\beta_n)$; ex. gr. $\beta_{11} = 48 + \frac{1}{2}.48 = 72$, itemque $\beta_{10} \backsim \beta_8 + \frac{1}{2}\beta_8 \backsim 33 + \frac{33}{2} \backsim 33 + \frac{33+149}{2} \backsim 33 + 91 \backsim 124$. Hoc modo seriem continuando inveniuntur schemata: $\begin{bmatrix} XV+ \\ .13 \end{bmatrix}, \begin{bmatrix} XIX(+) \\ .\ \ -8 \end{bmatrix} = \begin{bmatrix} XIX- \\ .8 \end{bmatrix}, \begin{bmatrix} XXXI+ \\ .\ \ 2 \end{bmatrix}$, commoda quidem, attamen nimis magnos numeros requirentia. Huic igitur rei interdum haud medetur, nisi schemata tertii generis adhibeantur.

III) Demum igitur schematis generalis $\begin{bmatrix} +\nu \\ \alpha.\beta \end{bmatrix}$ numeri auxiliares α, β, et ν, dato divisori (p) competentes, perquirendi sunt.

Primum igitur, si alteruter numerorum α, β datur, reliqui modis nuper edoctis invenientur.

$$149)300..(2 \quad \frac{298}{2}$$

Sit ex. gr. $p = 149$ et $\alpha = 3$. Dividatur 300 per 149, et residuum 2 pro β accipiatur, exsistente $\nu = 2$ (= numero cifrarum 00). Sin vero datum fuerit $\beta = 2$, divisio reverso ordine instituatur. Schema igitur divisoris 149 erit

$$\left\{ \begin{array}{r} \beta = +2 \\ +298 \\ \hline \alpha = 3 \mid 00 \\ \mid \nu \end{array} \right\}$$

$\begin{bmatrix} +II \\ 3.2 \end{bmatrix}$. Sin vero exponens ν datur, (ut sæpissime accidit, quandoquidem præcavendum sit, ut nimis magnus evadat), resolvenda est æquatio formæ $ax + by = cz$ in numeris integris minimis. Quæritur enim numerus formæ $\beta \cdot 10^\nu - \alpha$ multiplus divisoris dati ($p = c$), ideoque $\beta \cdot 10^\nu - \alpha = mp$, seu $\alpha = \beta \cdot 10^\nu - mp$; unde $\alpha \underset{p}{\backsim} \beta \cdot 10^\nu$, seu $\frac{\alpha}{\beta} \underset{p}{\backsim} 10^\nu$. Sit vero $10^\nu \underset{p}{\backsim} \gamma$, et quæritur fractio $\begin{bmatrix} \frac{\alpha}{\beta} \end{bmatrix}$ in numeris minimis ipsi γ congrua, quam interdum abbreviandi quodam genere invenire licet. Sit ex. gr. $\gamma = 30$ et $p = 53$, et scribatur $\frac{\alpha}{\beta} \underset{53}{\backsim} 30$, seu $\frac{\alpha}{\beta} \underset{53}{\backsim} \frac{30.16}{16}$; at $30 \cdot 16 = 480 \underset{53}{\backsim} 3$, ideoque $\frac{\alpha}{\beta} \underset{53}{\backsim} \frac{3}{16}$, seu $\frac{\alpha}{\beta} \backsim \frac{3-53}{16} \backsim \frac{-25}{8} \backsim \frac{28}{8} \backsim \frac{7}{2}$; quare accipi potest $\alpha = 7$ et $\beta = 2$.

Generatim vero æquatio $ax + y = bz$ resolvitur, idque in numeris minimis, (in integris enim, nullo negotio habetur $y = bz - ax$, exsistentibus z et x integris quibusvis), hunc in modum: Quæratur vulgari modo (nempe divisione continua) maximus communis di-

visor numerorum a et b, itemque fractiones $\frac{\alpha_0}{\beta_0}$, $\frac{\alpha_1}{\beta_1}$, $\frac{\alpha_2}{\beta_2}$ &c.. ad $\frac{a}{b}$ convergentes, exsistentibus residuis correspondentibus r_0, r_1, r_2 &c.; eritque universim $\pm r = y$, $z = \alpha$ atque $x = \beta$.

Sit. ex. gr. æquatio data $y = 1000\,z - 113\,x$, et invenientur

quoti: 8, 1, 5, 1, 1, 1,

unde fractiones convergentes $\frac{1}{8}, \frac{1}{9}, \frac{6}{53}, \frac{7}{62}, \frac{13}{115}, \frac{20}{177}$,

cum residuis... 96, 17, 11, 6, 5, 1.

Ex notissima vero fractionum ad eandem rationem convergentium indole est

(resid. $=\alpha$) quoti	$(\beta) =$ (numeratori)
113) 1000 (8	0 : 1
96 : 96 = 1	1 : 8
17) 85 (5	1 : 9
11 : 11 = 1	6 : 53
6) 6 (1	7 : 62
5 : 5 = 1	13 : 115
1	20 : 177

$1000 - 8 \,.\, 113 = 96 (= r_0)$, $1000 \,.\, 1 - 113 \,.\, 9 = -17$, $1000 \,.\, 6 - 113 \,.\, 53 = 11$ &c.., ideoque æquationis $y = 1000 \,.\, z - 113 \,.\, x$ solutiones:

$z =$	1	1	6	7	13	20
$x =$	8	9	53	62	115	177
$y =$	$+96$	-17	$+11$	-6	5	-1

Quoniam vero schema formæ $\binom{III}{\alpha \,.\, \beta}$ divisoris 113 æquationem $\alpha = \beta \,.\, 10^3 - 113 \,.\, p$ solvendam postulat, erit

$$\alpha = y = 96,\ -17,\ +11,\ -6,\ +5,\ -1$$

$$\text{atque } \beta = z = 1,\quad 1,\quad 6,\quad 7,\quad 13,\quad 20;$$

quorum tamen valorum nonnulli tantum idonei videntur, et schemata subministrant: $\binom{III+}{11.6}$, $\binom{III-}{6.7}$, $\binom{III-}{1.20}$ seu $\binom{III-}{.20}$ seu $\binom{IV+}{.\ 2}$; (littera enim β, cifra diminuta, seu per 10 divisa, exponentem unitate auget; α vero minuit). Universim igitur ad schema $\binom{\nu}{\alpha \,.\, \beta}$ inveniendum, divisori dato (p) competens, expo-

nens ν successive $= 2$ (si $p < 10^2$), 3, 4 &c. ponendus est, et maximus quasi communis divisor ipsorum p et 10^ν quærendus, idque continua divisione, et quidem alternis sub formis: $\left[\begin{matrix} p) & 10^\nu & (\text{quotus } (q) \\ & \text{resid. } r & \end{matrix}\right]$ atque $[p : r = q_,]$, (ut in exemplo modo computato vidisti); deindeque numeratores fractionum convergentium ab origine $0 : 1$ vulgari modo (nempe continue ducendo in quotos $q_1\, q_2$ &c. et addendo) quærendi sunt, et hi ipsi valores ipsius β præbebunt, residua vero valores ipsius α, signis cum divisionis forma alternantibus, nempe $+$ ad residuum ex prima forma, $-$ vero ex secunda. Sit ex. gr. $p = 179$, $\nu = 3 = III$. Computando ut diximus et a latere *) vides, et eligendo habebimus schemata: $\binom{III-}{12.5}$, $\binom{III+}{7.12}$, $\binom{III-}{5.17}$, ea tamen non commodissima. Capiamus igitur $\nu = 4$; et simili computo effecto **), habebimus schema $\binom{-IV}{2.15}$, quod duplicatum (numeri enim α et β in eundem numerum ad p primum ducere licet), suppeditat $\binom{-IV}{4.30}$, seu $\binom{-V}{4.3}$ schema praxi maxime idoneum.

Exempla adhuc nonnulla, exercitii caussa, computata apponimus.

*)

$-\alpha$	$+\alpha$	β
179)	1000(5	0
—	105 = 1	1
74)	(1	1
—	31 = 2	2
12)	(2	5 (= 2.2 + 1)
—	7 = 1	12 (= 2.5 + 2)
5)	(1	17 (= 1.12 + 5)
	2	29

**)

$-\alpha$		(β)
179)	10000(55	0
	1050:	
—	: 155 = 1	1
24)	(6	1
	: 11 = 2	7
2		15